# A Tradable Permit System in an Intertemporal Economy

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### Abstract

Tradable permit schemes are market-based instruments that many countries can potentially use to improve the management of their natural resources and to reduce pollution emissions. This study analyzes the intertemporal allocation of tradable permits, including the founding of a bank of permits. We show that Tinbergen's policy principle applies to the intertemporal permit system: at least as many policy instruments are required as policy objectives. The policy instruments are required to have the same Lebesgue measure in a continuous time model. Thus, the merits of the artificial market system analyzed by Montgomery (1972) are reduced in an intertemporal setup.

Keywords: Tradable permits; Intertemporal arbitrage condition; Knowledge updating; Tinbergen's policy principle; Artificial market.JEL Classification: H23; K32; Q58.

### 1 Introduction

The creation of an artificial market for public goods/bads using tradable permits is gaining popularity among policymakers as an environmental policy option. A number of studies have analyzed tradable permits from several aspects: the spatial allocation in a competitive economy ([13]); under imperfect competition ([7]); with transaction costs ([15]); compared with other policy instruments in an intertemporal competitive economy ([18]); and in the presence of uncertainty à la Weitzman's "quantity vs. price" ([10]).

Recently, the intertemporal allocation of tradable permits has attracted attention. This reflects the fact that most existing emissions trading programs have permitted banking, i.e., the transfer of unused permits into future commitment periods ([2]). On the other hand, borrowing, i.e., the use of permits that were originally designated to a later period, is rarely applied formally in practice. However, in reality it is allowed with a penalty imposed on defaulters. For instance, in the Kyoto Protocol of the United Nations' Framework Convention on Climate Change, if a party's emissions have exceeded its assigned amount in the first period, it is required to make up the difference between its allowed and actual emissions plus an additional deduction of 30 percent in the next period.

This paper contributes to the theory on the intertemporal tradable permit system, referred to as a *bankable permit system*. The early studies ([6], [17]) showed that the introduction of banking and borrowing enables firms to achieve a cumulative emission target over a fixed planning horizon at the lowest discounted cost. However, subsequent studies ([11],[12], [20], [9]) found that the equilibrium outcome may not necessarily coincide with the socially optimal emission path, and that such a permit system may fail to achieve efficient pollution control in a competitive economy.

These findings are summarized as a lack of an appropriate intertemporal arbitrage condition. Consider a holder of permits with amount B > 0. Let r(t) be the equilibrium interest rate at time  $t \ge 0$ . For any  $t, t' \ge 0$  in the valid term of the permits, the equilibrium price of the permits  $p^e(t)$  satisfies the arbitrage condition:

$$p^{e}(t)B = p^{e}(t')B\exp\left(-\int_{t}^{t'}r(s)ds\right).$$

As the choice of the initial value  $p^e(0)$  is arbitrary, depending on the choice of the numéraire, this equation determines the path of equilibrium prices entirely. On the other hand, the *efficient*  prices of permits  $p^*(t)$  are determined by the marginal damage cost of pollution emissions at the social optimum,  $p^*(t) = MC(t)$ .<sup>1</sup> Because this efficient pricing rule is independent of the arbitrage condition, there is no reason for the equilibrium prices and the efficient prices to have the same values.

The condition under which these two sets of prices coincide can be shown easily by replacing B with B(t). Setting  $p^e(t) = p^*(t)$ , we have:

$$p^{*}(t)B(t) = p^{*}(t')B(t')\exp\left(-\int_{t}^{t'}r(s)ds\right).$$

By differentiating this with respect to t' and evaluating at t' = t, we have:<sup>2</sup>

$$\frac{\dot{B}(t)}{B(t)} = r(t) - \frac{\dot{p}^*(t)}{p^*(t)}.$$
(1)

The left-hand side  $\dot{B}/B$  is the rate of return from holding a permit. The equation implies that the equilibrium price coincides with the efficient price only if the rate of return is equal to  $r - \dot{p}^*/p^*$ . Notice that the equation is a very familiar intertemporal arbitrage condition: the sum of the return and capital gain of an asset coincides with the interest rate at each point in time,  $\dot{B}/B + \dot{p}/p = r$ .

The existing literature assumes no return from holding permits (B = 0), reflecting the practice of tradable permit schemes, and finds the problem of an inefficient intertemporal allocation of pollution exists. In order for a bankable permit system to implement efficient pollution control, a regulatory authority needs to exogenously set the appropriate rates of return of the permits (the permit interest rates), defined in (1). However, a problem of information collection arises for the regulatory authority: the exact calculation of permit interest rates requires knowledge on the entire equilibrium paths of the efficient permit prices  $p^*(t)$  and the market interest rates r(t). This could represent a heavy burden, greatly reducing the merit of a tradable permit system as a policy option for pollution control.

Recognizing these problems, we intend to analyze a mechanism by which the permit interest rates are determined by a market without additional policy instruments. The mechanism involves adding a bank of permits to a bankable permit system. Each permit holder has an account of permits in the bank. Deposits and withdrawals are made in terms of permits, whereas the balance

<sup>&</sup>lt;sup>1</sup>For the case of cost-effective pollution allocation, MC(t) refers to the minimum marginal cost of pollution abatement.

 $<sup>^2 {\</sup>rm Throughout}$  the paper, we suppress the expression "almost everywhere" unless confusion could arise.

is expressed in monetary terms by multiplying the quantity of permits held by their market price. The prevailing market interest rate is applied. It is shown that this mechanism achieves efficient pollution control at an equilibrium. However, another problem arises: it also allows other undesirable equilibria. A market may not work well for the adjustment of the arbitrage condition of permits. This indeterminacy can be a serious problem for regulatory authorities because one can no longer ensure any definite outcome.

Imperfect knowledge makes the problems more complicated. It is true that our knowledge on pollution damage is imperfect and updated intermittently. We incorporate this fact and show that the regulatory authority needs a further instrument, namely intermittent devaluation of the permits.

We argue that these difficulties can be related to Tinbergen's long-standing policy principle: at least as many policy instruments are needed as policy targets (Tinbergen, [19]). The intertemporal permit market needs to achieve two objectives, environmental regulation and minimization of the costs of the regulation. The provision of a fixed number of permits achieves only the latter. To accomplish the former objective, the regulatory authority needs to set the permit interest rates and, at times, to devalue permits. This requirement does not appear in a static model. Therefore, we conclude that in an intertemporal setting, the merits of a tradable permit system, or Montgomery's artificial market ([13]), are reduced.

The rest of this paper is organized as follows. Section 2 formally states the problem of the arbitrage condition. Section 3 analyzes the proposal of a bankable permit system with a permit bank and examines its indeterminate equilibria. In Section 4, we consider imperfect knowledge on pollution damage. Section 5 discusses the implications of the results from a practical viewpoint.

### 2 The Problem of the Arbitrage Condition

### 2.1 Welfare maximization

Consider a simple competitive economy. There is a continuum of identical infinitely lived households with population (the Lebesgue measure) one. Let  $\rho > 0$  be the time discount rate and let u(c, X) be the instantaneous utility function, where c denotes the consumption rate and X denotes the pollution flow. Assume that  $u : \mathbb{R}^2_+ \to \mathbb{R} \cup \{-\infty\}$  is a strictly concave and smooth function with  $u_c(\tilde{c}, \tilde{X}) > 0$ ,  $u_X(\tilde{c}, \tilde{X}) < 0$ ,  $\lim_{c\to 0} u_c(c, \tilde{X}) = \infty$ , and  $\lim_{X\to 0} u_X(\tilde{c}, X) = 0$  for all  $(\tilde{c}, \tilde{X}) \in \mathbb{R}^2_{++}$ .<sup>3</sup> The production technology of the economy is described by a "pollution as input" production function F(K, X), where  $K \ge 0$  is the capital stock. We assume that the production function  $F : \mathbb{R}^2_+ \to \mathbb{R}_+$  is linear homogeneous, concave, and smooth with F(0, X) = F(K, 0) = 0 for all  $K \ge 0$  and  $X \ge 0$ . There is an upper limit on pollution emissions of X because pollution is in fact a byproduct of the production, not an input. Assume that the upper limit proportionally depends on the amount of the capital stock K. Then, letting  $\phi > 0$ , the effective domain of the production function is defined by  $A := \{(K, X) \in \mathbb{R}^2_+ | X \in [0, \phi K]\}$ . Let  $K_0$  be the initial endowment of capital stock. We assume no capital depreciation for simplicity.

The social planner's problem is formulated as:

$$\max_{c(t),X(t)} \int_0^\infty u(c(t),X(t))e^{-\rho t}dt$$
(2)

subject to 
$$\dot{K}(t) = F(K(t), X(t)) - c, \ (K(t), X(t)) \in A, \ K(0) = K_0 > 0.$$
 (3)

The associated Hamiltonian is given by:

$$H(c, X, K, \lambda, \mu) = u(c, X) + \lambda [F(K, X) - c] + \mu \left(\phi K - X\right).$$

If  $(c^*(t), X^*(t), K^*(t))$  is an optimal path, then there exists  $\mu^*(t) \ge 0$ , with which (3) and the following equations hold for each  $t \ge 0$ :

$$u_X(c^*(t), X^*(t)) + u_c(c^*(t), X^*(t))F_X(K^*(t), X^*(t)) - \mu^*(t) = 0,$$
(4)

$$F_K(K^*(t), X^*(t)) - \rho + \phi \mu^*(t) / u_c(c^*(t), X^*(t)) = (d/dt) \ln u_c(c^*(t), X^*(t)),$$
(5)

$$\mu^*(t) \ge 0$$
, and  $\mu^*(t)[\phi K^*(t) - X^*(t)] = 0.$  (6)

Our interest is in how the optimal path  $(c^*(t), X^*(t), K^*(t))$  can be mimicked in a competitive economy when the regulatory authority implements a bankable permit system. Throughout this paper, it is assumed that the permit is emission-based, so that one unit of emission is allowed in return for one permit. Let B > 0 denote the total amount of permits distributed to each firm once at

<sup>&</sup>lt;sup>3</sup>The notation  $f_x$  for a function f denotes the partial derivative with respect to its argument x:  $f_x := \partial f / \partial x$ .

the initial time. We denote the rates of return for saving permits by a(t), a measurable function on  $[0, \infty)$ . A bankable permit system is defined by a pair (B, a(t)). Let the products be the numéraire good and let p(t) denote the price of the tradable permits at time t. r(t) is the interest rate, which satisfies  $r(t) = F_K(K(t), X(t))$  at equilibrium. As the production function is linear homogeneous, we can treat the production sector as being comprised of a single price-taking profit-maximizing firm. The firm's problem is given by:

$$\pi = \max_{K(t), X(t), y(t)} \int_0^\infty \left[ F(K(t), X(t)) - r(t)K(t) - p(t)y(t) \right] \exp\left(-\int_0^t r(s)ds\right) dt$$
(7)  
subject to  $\dot{B}(t) = a(t)B(t) - X(t) + y(t), \ B(0) = B,$   
 $K(t) \ge 0, X(t) \in [0, \phi K(t)], \ \text{and} \ \liminf_{t \to \infty} B(t) \exp\left(-\int_0^t a(s)ds\right) \ge 0,$ 

where y(t) is the amount of permits purchased (if y(t) > 0) or sold (if y(t) < 0) at time t, and the last constraint is a no-Ponzi game condition, without which the firm can buy permits unlimitedly and emit its pollution arbitrarily. The representative household's problem is:

$$\max_{c(\cdot)\geq 0} \int_0^\infty u(c(t), \tilde{X}(t)) e^{-\rho t} dt$$
subject to  $\dot{A}(t) = r(t)A(t) - c$ ,  $A(0) = K_0 + \tilde{\pi}$ ,
$$\lim_{t \to \infty} \inf A(t) \exp\left[-\int_0^t r(s) ds\right] \geq 0,$$
(8)

where X(t) is the pollution flow that is exogenous for a household, A(t) is the balance of the assets, and  $\tilde{\pi}$  is the profit distribution from firms. The last inequality in the constraint is a no-Ponzi- game condition.

**Definition 1**: Competitive equilibrium. The tuple

$$(c^{e}(t), X^{e}(t), y^{e}(t), \tilde{X}^{e}(t), K^{e}(t), \pi^{e}, \tilde{\pi}^{e}, A^{e}(t), B^{e}(t); r^{e}(t), p^{e}(t); K_{0}, B, a(t))$$

is a competitive equilibrium if it satisfies the following conditions:

- 1.  $(X^{e}(t), y^{e}(t), K^{e}(t), B^{e}(t))$  is a solution for the firm's problem (7) given  $(r^{e}(t), p^{e}(t); B, a(t))$ .
- 2.  $(c^{e}(t), A^{e}(t))$  is a solution for the household's problem (8) given  $(K_0, \tilde{\pi}^{e}, \tilde{X}^{e}(t), r^{e}(t))$ .

3. 
$$c^{e}(t) = F(K^{e}(t), X^{e}(t)) - K^{e}(t), X^{e}(t) = \tilde{X}^{e}(t), y^{e}(t) = 0, \pi^{e} = \tilde{\pi}^{e}.$$

Now, we formally state the necessary conditions under which a bankable permit system (B, a(t)) can achieve the social optimum in a competitive economy.

**Proposition 1** : The competitive equilibrium,

$$(c^{e}(t), X^{e}(t), y^{e}(t), \tilde{X}^{e}(t), K^{e}(t), \pi^{e}, \tilde{\pi}^{e}, A^{e}(t), B^{e}(t); r^{e}(t), p^{e}(t); K_{0}, B, a(t))$$

is optimal only if the bankable permit system satisfies:

$$a(t) = \rho - \frac{d}{dt} \ln \left[ -u_X(c^*(t), X^*(t)) \right], \text{ and}$$
(9)

$$B = \limsup_{T \to \infty} \int_0^T X^*(t) \exp\left[-\int_0^t a(s)ds\right] dt.$$
 (10)

**Proof.** Suppose that an equilibrium path is optimal, that is,  $(c^e(t), X^e(t), K^e(t)) = (c^*(t), X^*(t), K^*(t))$ . The (current value) Hamiltonian for the firm's problem (7) is written as:

$$H(K, X, y, B, \tilde{\lambda}, \tilde{\mu}, t) = F(K, X) - r(t)K - p(t)y + \tilde{\lambda} \left[a(t)B - X + y\right] + \tilde{\mu} \left(\phi K - X\right).$$

Therefore, at equilibrium, the following hold:

(a) 
$$F_K(K^*(t), X^*(t)) - r(t) + \tilde{\mu}(t)\phi = 0$$
; (b)  $F_X(K^*(t), X^*(t)) - p(t) - \tilde{\mu}(t) = 0$ ; (11)  
(c)  $\dot{p}(t) = [r(t) - a(t)] p(t)$ ; (d)  $\tilde{\mu}(t) (\phi K^*(t) - X^*(t)) = 0, \tilde{\mu}(t) \ge 0$ .

The no-Ponzi game condition in (7) should be satisfied with equality, so that the transversality condition is satisfied:

$$\liminf_{t \to \infty} B(t) \exp\left(-\int_0^t a(s)ds\right) = p(0)^{-1}\liminf_{t \to \infty} p(t)B(t) \exp\left(-\int_0^t r(s)ds\right) = 0,$$
(12)

where the first equality follows from (11c). For the household, the Keynes–Ramsey rule (the Euler equation) holds at equilibrium:

$$\rho - r(t) = (d/dt) \ln u_c(c^*(t), X^*(t)).$$
(13)

The no-Ponzi game condition in (8) should be satisfied with equality, so that the transversality condition is satisfied:

$$\liminf_{t \to \infty} A(t) \exp\left[-\int_0^t r(s)ds\right] = u_c(c^*(0), X^*(0))^{-1} \liminf_{t \to \infty} u_c(c^*(t), X^*(t))A(t)e^{-\rho t} = 0, \quad (14)$$

where the first equality follows from (13). These equations (11), (12), (13), and (14) and Condition 3 in Definition 1 fully characterize the equilibrium path. From (5), (11a), and the Keynes–Ramsey rule (13), we have  $\tilde{\mu}(t) = \mu^*(t)/u_c(c^*(t), X^*(t))$ . Then, by (4) and (11b):

$$p(t) = -u_X(c^*(t), X^*(t)) / u_c(c^*(t), X^*(t)).$$
(15)

Using (11c), (13), and (15), we have (9). The necessity of (10) follows from the state equation in (7) and (12):

$$\liminf_{t \to \infty} \left\{ B - \int_0^t X^*(\tau) \exp\left[ -\int_0^\tau a(s) ds \right] d\tau \right\} = \liminf_{t \to \infty} \left\{ B(t) \exp\left[ -\int_0^t a(s) ds \right] \right\} = 0.$$

The following three comments are provided in relation to the proposition:

1. As indicated in the Introduction, a bankable permit system is a useful policy tool only if the rate of return of permits (the permit interest rate) is set appropriately, as in (9) of Proposition 1.

2. We obtain the results in a general equilibrium framework, whereas Leiby and Rubin ([12]) obtained similar results in a partial equilibrium model (economic surplus maximization).

3. Even though no abatement is optimal  $(X^*(t) = \phi K^*(t))$ , the price of permits is positive, as shown in (15). This indicates that even when pollution *control* is not required, pollution *pricing* is necessary—otherwise, some markets would be distorted. The principle is that pollution should be priced at the aggregate marginal damage at an efficient equilibrium, which is demonstrated for the Pigovian tax in a general equilibrium model by Baumol and Oates ([1, Chapter 4]). Hence, a permit system should be implemented over all periods, no matter what the level of pollution control is. Stokey ([18]) discovered this fact and showed that without pricing of pollution, the capital market is distorted so that it prevents an efficient outcome. Notice that this point does not appear in a static analysis or in a partial equilibrium analysis.

#### 2.2 Cost minimization

We consider the cost minimization problem to attain the targeted pollution control level. Obviously, this problem contains the social planner's problem in the previous subsection. The aim of this subsection is to generalize the results in Proposition 1 to a cost-effective pollution control.

Let  $[0, T), T \in \mathbb{R}_{++} \cup \{\infty\}$  denote the planning horizon of a tradable permit scheme, which may be finite or infinite. The environmental objective for the total pollution emission flow is exogenously determined and denoted by a function of time  $X^* : [0, T) \to \mathbb{R}_+$ , which has the following properties:

**Assumption 1**: 
$$X^*(t) \in \mathbb{C}^1$$
, and  $0 < \inf \{X^*(t) | t \in [0, T)\} \le \sup \{X^*(t) | t \in [0, T)\} < \infty$ , (16)

where  $\mathbb{C}^k$  means the space of k times continuously differentiable functions. We allow heterogeneity of technologies among firms, although we continue to assume that the firms act as price takers. The regulatory authority provides permits of the amount  $B_i$  for firm *i* at the initial period.<sup>4</sup> A bankable permit scheme is expressed by a quartet of planning horizon, target pollution control, permit distribution, and permit interest rates:

$$(T, X^*, (B_i)_{i=1}^n, a),$$
 (17)

where  $n(<\infty)$  is the number of firms that participate in the trading of the permits, and  $a:[0,T) \to \mathbb{R}$  is the permit interest rate, as before.

Let  $x_i$  be the pollution flow emitted by firm i, and let  $C^i(x_i, t)$  be its abatement cost. Assume

$$B_i = \int_0^T g_i(t) \exp\left[-\int_0^t a(s)ds\right] dt + \sum_{j \in J_i} G_i(t_j) \exp\left[-\int_0^{t_j} a(s)ds\right].$$

<sup>&</sup>lt;sup>4</sup>Alternatively, firm *i* is provided with the permits as a stock and/or as a flow during the planning horizon. This is described by a pair  $(g_i(t), \{G_i(t_j) | j \in J_i\})$ , where  $g_i : [0, T) \to \mathbb{R}_+$  is the flow of permits,  $G_i(t_j)$  is the amount of stock given at time  $t_j \in [0, T)$ , and  $J_i \subset \{1, 2, ...\}$  is a subset of the index set, possibly empty, finite, or infinite. This distribution scheme is the same as the lump sum grant:

that  $C^i: \mathbb{R}_+ \times [0,T) \to \mathbb{R}_+ \cup \{\infty\}$  has the following properties for each *i* and *t*:<sup>5</sup>

Assumption 2: 
$$C^i \in \mathbb{C}^2, C^i_{xx} > 0, \lim_{x \to 0} C^i_x(x, t) = -\infty, \text{ and } \exists \tilde{x}_i(t) > 0 : C^i(\tilde{x}_i(t), t) = 0.$$
 (18)

The social cost minimization problem is given as follows. For each  $t \in [0, T)$ :

$$C(X^*(t), t) = \min_{(x_i)_{i=1}^n} \sum_{i=1}^n C^i(x_i, t) \quad \text{subject to } (x_1, \dots, x_n) \in \mathbb{R}^n_+ \text{ and } \sum_{i=1}^n x_i \le X^*(t).$$
(19)

As the problem is to minimize a strictly convex function on a compact domain, the solution  $x_i^*(X^*(t), t)$ exists and is unique for each  $i \in \{1, 2, ..., n\}$ , each  $X^*(t) \ge 0$ , and each  $t \in [0, T)$ . Notice that for each  $i \in \{1, 2, ..., n\}$  and each  $t \in [0, T)$ , the function  $x_i^*(\cdot, t) : \mathbb{R}_{++} \to \mathbb{R}_{++}$  is continuously differentiable by the implicit function theorem. Therefore, the social cost function for pollution abatement  $C(\cdot, t)$ is also continuously differentiable. We define the associated shadow price as:

$$p^{*}(t) := -C_{X}(X^{*}(t), t) = -C_{x}^{i}(x_{i}^{*}(X^{*}(t), t), t), \text{ all } i \in \{1, 2, ..., n\}.$$
(20)

We are interested in how the efficient allocation of pollution emissions can be implemented by a bankable permit system. The market conditions surrounding a firm are represented by a pair (p(t), r(t)), where p(t) is the competitive price of permits and r(t) is the interest rate, as before. The cost minimization problem for individual firm *i* is given by:

$$\min_{x(t)\geq 0, y(t)} \int_0^T \exp\left[-\int_0^t r(s)ds\right] \left[C^i(x(t), t) + p(t)y(t)\right]dt$$
subject to  $\dot{B}(t) = a(t)B(t) - x(t) + y(t),$ 

$$B(0) = B_i, \ \liminf_{t \to T} B(t) \exp\left[-\int_0^t a(s)ds\right] \geq 0.$$
(21)

where  $y_i(t)$  is the quantity of permits bought  $(y_i(t)>0)$  or sold  $(y_i(t)<0)$ .<sup>6</sup> The permit market

$$C^{i}(x,t) = \left[F(K(r(t)), \phi K(r(t))) - r(t)K(r(t))\right] - \max\left\{F(K,X) - r(t)K|K \ge 0\right\}.$$

 $^{6}$ Instead of the no-Ponzi game condition (the last inequality in the constraint of (21)), some studies have incorporated the nonnegativity condition, assuming that the borrowing of permits is not allowed. This choice affects the feasible set of emission paths, but makes no difference to the results as long as the set contains the efficient emission path.

<sup>&</sup>lt;sup>5</sup>Using the notation in the previous subsection and letting  $K(r) := \arg \max \{F(K, \phi K) - rK | K \ge 0\}$ , the abatement cost function is defined as:

equilibrium is defined as follows:

**Definition 2**: Permit market equilibrium. Let  $((B_i)_{i=1}^n, a(t))$  and r(t) be given. The tuple

$$((x_i^e(t), y_i^e(t))_{i=1}^n; p^e(t), r(t); (B_i)_{i=1}^n, a(t))$$

on [0, T) is a permit market equilibrium if the following two conditions are satisfied:

- 1.  $\sum_{i=1}^{n} y_i^e(t) = 0$  for each  $t \in [0, T)$ .
- 2. For each  $i \in \{1, ..., n\}$ , pair  $(x_i^e(t), y_i^e(t))$  solves Problem (21), given  $(p^e(t), r(t); (B_i)_{i=1}^n, a(t))$ .

Then, we have the following result:

**Proposition 2** : Let  $(T, X^*, (B_i)_{i=1}^n, a)$  be a bankable permit system. With given r(t), a permit market equilibrium  $((x_i^e(t), y_i^e(t))_{i=1}^n; p^e(t), r(t); (B_i)_{i=1}^n, a(t))$  achieves the efficient allocation of pollution emissions,  $x_i^e(t) = x_i^*(X^*(t))$ , if and only if the bankable permit system satisfies:

$$a(t) = r(t) - \dot{p}^{*}(t)/p^{*}(t), and$$
(22)

$$\sum_{i=1}^{n} B_i = \limsup_{\mathcal{T} \to \mathcal{T}} \int_0^{\mathcal{T}} X^*(t) \exp\left[-\int_0^t a(s) ds\right] dt < \infty,$$
(23)

**Proof.** The following two statements are equivalent. (i) Pair  $(x_i^e(t), y_i^e(t))$  is the solution to Problem (21). (ii) Pair  $(x_i^e(t), y_i^e(t))$  satisfies:

(a) 
$$p^{e}(t) = -C_{x}^{i}(x_{i}^{e}(t), t)$$
, (b)  $\dot{p}^{e}(t)/p^{e}(t) = r(t) - a(t)$ , (24)  
(c)  $\liminf_{\mathcal{T} \to T} p^{e}(T)B(T) \exp\left[-\int_{0}^{\mathcal{T}} r(t)dt\right] = 0.$ 

Here, the necessity of the transversality condition (24c) follows from the no-Ponzi game condition in Problem (21) and the positivity of the permit price shown by (24 a) ("only if" part). Suppose  $x_i^e(t) = x_i^*(X^*(t))$ . Then, (22) is immediately obtained from (20) and (24 a, b). The state equation and the no-Ponzi game condition in Problem (21) yield:

$$B_{i} - \limsup_{\mathcal{T} \to \mathcal{T}} \int_{0}^{\mathcal{T}} \left[ x_{i}^{*}(X^{*}(t)) - y_{i}^{e}(t) \right] \exp\left[ -\int_{0}^{t} a(s) ds \right] dt = 0.$$
(25)

We sum up (25) over  $i \in \{1, 2, ..., n\}$  to obtain (23) ("if" part). Suppose that the bankable permit system satisfies (22) and (23). Then, from (22) and (24 b), there is a real number k > 0 with which  $p^e(t) = kp^*(t)$ . Then, (20) and (24 a) imply  $C_x^i(x_i^e(t), t) = kC_x^i(x_i^*(t), t)$ . Suppose that k > 1. Then,  $x_i^e(t) > x_i^*(t)$  for almost all t and all i by the strict convexity of  $C^i(\cdot, t)$ . This implies that:

$$\limsup_{\mathcal{T}\to T} \sum_{i=1}^{n} \int_{0}^{\mathcal{T}} \left[ x_{i}^{e}(t) - y_{i}^{e}(t) \right] \exp\left[ -\int_{0}^{t} a(s) ds \right] dt > \sum_{i=1}^{n} B_{i}$$

Then, there exists a firm *i* that does not satisfy its no-Ponzi game condition. In the case where k < 1, we have the opposite inequality above and, thus, there remain some permits at the end of the planning horizon, which contradicts the optimality condition (24 c). Therefore, k = 1 and  $x_i^e(t) = x_i^*(t)$ . The efficient equilibrium is constructed by choosing  $(y_i^e(t))_{i=1}^n$  so as to satisfy the no-Ponzi game condition and Condition 1 in Definition 2.

In the case of both socially optimal and cost-effective pollution controls, a bankable permit system needs to include an appropriate set of permit interest rates,  $a(t) = r(t) - \dot{p}^*(t)/p^*(t)$ . However, this requires that the regulatory authority knows in advance the shadow prices for the pollution control  $p^*(t)$  as well as the equilibrium path of interest rates r(t) over the planning horizon.

One exception is when the time-aggregated total pollution matters. That is, where the regulatory authority is not concerned about pollution control at each point in time, but about total emission control over time. Then, Hotelling's rule is applied to the total pollution and we have a(t) = 0(see [17], [11]). Except for this case, the burden of information collection would greatly reduce the merit of a bankable permit system compared with other policy tools. A Pigovian tax needs only the shadow price  $p^*(t)$  at each point in time. A one-period (perishable) permit system is much easier: the regulatory authority needs only to issue the permits with amount  $X^*(t)$  at each point in time, which works as well as the Pigovian tax and does not distort markets, unlike direct regulation (see Stokey[18]).

### 3 Trading with a Permit Bank

In this section, we analyze a bankable permit system without setting the exogenous rates of return for holding permits. We start with the individual minimization problem (21) in terms of money. Let Q(t) denote the monetary value of the permits held by firm *i* at time *t* and let  $Q_i^o$  be the initial value, i.e.,  $Q_i^o = p(0)B_i$ . Correspondingly, the state equation in Problem (21) is expressed in monetary units by:

$$\dot{Q}(t) = r(t)Q(t) - p(t)x(t) + p(t)y(t).$$

Then, we have the following problem:

$$\min_{x(t)\geq 0, y(t)} \int_0^T \exp\left[-\int_0^t r(s)ds\right] \left[C^i(x(t)) + p(t)y(t)\right]dt$$
(26)
subject to  $\dot{Q}(t) = r(t)Q(t) - p(t)x(t) + p(t)y(t),$ 

$$Q(0) = Q_i^o, \quad \liminf_{t \to T} Q(t) \exp\left[-\int_0^t r(s)ds\right] \geq 0.$$

Notice that with  $a(t) = r(t) - \dot{p}(t)/p(t)$  and B(t) = Q(t)/p(t), Problem (26) becomes mathematically equivalent to Problem (21). However, unlike (21), the permit interest rates a(t) disappear in (26). Therefore, a bankable permit system can degenerate into  $(T, X^*, (B_i)_{i=1}^n)$ . Correspondingly, we modify the definition of permit market equilibrium as follows:

**Definition 3**: Degenerate permit market equilibrium. Let  $(B_i)_{i=1}^n$  and r(t) be given. The tuple

$$((x_i^e(t), y_i^e(t))_{i=1}^n; p^e(t), r(t); (B_i)_{i=1}^n)$$

on [0, T) is a degenerate permit market equilibrium if the following two conditions are satisfied:

- 1.  $\sum_{i=1}^{n} y_i^e(t) = 0$  for all  $t \in [0, T)$ .
- 2. For each  $i \in \{1, ..., n\}$ , pair  $(x_i^e(t), y_i^e(t))$  solves Problem (26) given  $(p^e(t), r(t); (B_i)_{i=1}^n)$ .

Then, we have the following proposition:

**Proposition 3** : If each firm's problem is given by (26), then a degenerate bankable permit system  $(T, X^*, (B_i)_{i=1}^n)$  with  $\sum_{i=1}^n B_i$  satisfying (22) and (23) implements the efficient allocation of pollution emissions,  $x_i^e(t) = x_i^*(X^*(t), t)$  at a degenerate permit market equilibrium.

**Proof.** Assume that the equilibrium price of permits is given by  $p^*(t)$  defined in (20). Then, follow the proof of Proposition 2 ("if" part).

A permit bank, which is a bankable permit system with a permit bank without a(t), can be established using the firm's problem in (26). In the bank, each permit holder has an account of permits. Deposits and withdrawals are made in terms of permits, whereas the balance is expressed in monetary terms by multiplying the quantity of permits held by the market price. The prevailing market interest rates are applied. With the bank, the permit interest rates are found subsequently from equilibrium interest rates and permit prices:  $a(t) = r(t) - \dot{p}^*(t)/p^*(t)$ . The permit bank moves the determination of a(t) from the regulatory authority to a market. However, this mechanism has flaws. First, it cannot avoid the information collection problem of a bankable permit system. Information on r(t) and  $p^*(t)$  is required to determine the amount of issued permits  $\sum_{i=1}^{n} B_i$ . Second, to make matters worse, we can show that it adds to the problem of multiple equilibria, so that the regulatory authority can no longer ensure an efficient outcome. First, we put forward the following lemma and then state the result. Let  $\hat{X} : [0, T) \to \mathbb{R}_{++}$  be a measurable function satisfying  $\hat{X}(t) <$  $\sum_{i=1}^{n} \tilde{x}_i(t)$ , where  $\tilde{x}_i$  is defined in Assumption 2.

**Lemma 1** : Suppose that a bankable permit system with a permit bank is implemented with the permit distribution  $(B_i)_{i=1}^n$ . If

$$\limsup_{\mathcal{T}\to \mathcal{T}} \int_0^{\mathcal{T}} C_X(\hat{X}(t), t) \hat{X}(t) \exp\left[-\int_0^t r(s) ds\right] dt = C_X(\hat{X}(0)) \sum_{i=1}^n B_i$$
(27)

holds, then there is a degenerate permit market equilibrium with permit prices  $\hat{p}^e(t) = -C_X(\hat{X}(t), t)$ .

**Proof.** Each firm *i* can satisfy its no-Ponzi game condition and transversality condition by choosing  $y_i^e(t) = \hat{y}_i^e$  such that:

$$\hat{y}_i^e = \frac{\hat{p}^e(0)B_i - \lim_{\mathcal{T} \to \mathcal{T}} \int_0^{\mathcal{T}} \hat{p}^e(t)x_i^e(t)\exp\left[-\int_0^t r(s)ds\right]dt}{\lim_{\mathcal{T} \to \mathcal{T}} \int_0^{\mathcal{T}} \hat{p}^e(t)\exp\left[-\int_0^t r(s)ds\right]dt}.$$
(28)

The pollution emission for firm *i* is chosen as  $x_i^e(t) = x_i^*(\hat{X}(t), t)$ , where  $x_i^*(\cdot)$  is implicitly defined by (20). Then, pair  $(x_i^e(t), y_i^e(t)) = (x_i^*(\hat{X}(t), t), \hat{y}_i^e)$  satisfies Condition 2 in Definition 3 of the degenerate bankable permit equilibrium. Condition 1,  $\sum_{i=1}^n \hat{y}_i^e = 0$ , follows from (27), (28), and  $\sum_{i=1}^n x_i^*(\hat{X}(t), t) = \hat{X}(t)$ .

**Proposition 4** : Let  $(T, X^*, (B_i)_{i=1}^n)$  be a permit market system with a permit bank. If there is a

time point  $\tau \in (0,T)$  such that:

$$X^{*}(\tau)C_{XX}(X^{*}(\tau),\tau)/C_{X}(X^{*}(\tau),\tau) \neq -1,$$
(29)

then there exists a continuum of degenerate permit market equilibria. Each equilibrium, except for one, fails to implement the target pollution control.

**Proof.** Notice first that (27) in Lemma 1 holds if  $\hat{X}(t) = X^*(t)$ , the target of pollution control. As can be understood from the following argument, without loss of generality we can assume that there is  $\tau \in (0,T)$  such that  $X^*(\tau)C_{XX}(X^*(\tau),\tau)/C_X(X^*(\tau),\tau) > -1$ . Then, by Assumptions 1 and 2, there is a sufficiently small positive number  $\delta$  and there is a time interval  $(\varepsilon_L, \varepsilon_U)$  containing  $t = \tau$  such that  $X^*(t)C_{XX}(X^*(t),t)/C_X(X^*(t),t) > -1 + \delta$  for  $t \in (\varepsilon_L, \varepsilon_U)$ . Picking up  $\tilde{\delta} \in (0, \delta)$ and a nondegenerate interval  $(\tilde{\varepsilon}_L, \tilde{\varepsilon}_U) \subset (\varepsilon_L, \varepsilon_U)$ , consider a total emissions path:

$$\tilde{X}(t) = \begin{cases} X^*(t) & \text{if } t \notin (\tilde{\varepsilon}_L, \tilde{\varepsilon}_U) \\ X^*(t) + \tilde{\delta} & \text{if } t \in (\tilde{\varepsilon}_L, \tau] \\ X^*(t) - \tilde{\delta} & \text{if } t \in (\tau, \tilde{\varepsilon}_U) \end{cases}$$

As  $d \left[ C_X(X^*(t), t) X^*(t) \right] / dX < 0$  on  $(\varepsilon_L, \varepsilon_U)$ , the following inequalities hold:

$$C_X(\tilde{X}(t),t)\tilde{X}(t) - C_X(X^*(t),t)X^*(t) \stackrel{<}{>} 0 \text{ if } t \in \begin{array}{c} (\tilde{\varepsilon}_L,\tau] \\ > & (\tau,\tilde{\varepsilon}_U) \end{array}$$
(30)

Therefore, we can choose interval  $(\tilde{\varepsilon}_L, \tilde{\varepsilon}_U)$  to satisfy:

$$\int_{\tilde{\varepsilon}_L}^{\tilde{\varepsilon}_U} \left[ C_X(\tilde{X}(t), t)\tilde{X}(t) - C_X(X^*(t), t)X^*(t) \right] \exp\left[ -\int_0^t r(s)ds \right] dt = 0.$$
(31)

Equation (31) implies that (27) holds with  $\hat{X}(t) = \tilde{X}(t)$ . Thus, by Lemma 1, we have a different equilibrium from the efficient one. Finally, notice that the inequalities in (30) imply that the set of intervals  $(\tilde{\varepsilon}_L, \tilde{\varepsilon}_U)$  satisfying (31) has the cardinality of continuum including the limit case of  $(\tilde{\varepsilon}_L, \tilde{\varepsilon}_U) = \emptyset$ .

The condition (29) in Proposition 4 is generic, and so is the statement. This proposition implies that when the regulatory authority oversees a market without setting permit interest rates, there arises indeterminacy owing to the existence of multiple equilibria. One policy instrument cannot achieve all the policy goals. We follow the implication of Tinbergen ([19]). That is, to implement an efficient pollution control in an intertemporal economy, at least as many policy instruments are required as policy objectives. In our model, the target  $X^*(t)$  has a measure  $\mathcal{L}([0,T))$ , where  $\mathcal{L}(\cdot)$ is the Lebesgue measure. That is why we need the instruments a(t) over the interval [0,T), the measure of which is the same,  $\mathcal{L}([0,T))$ .

### 4 Knowledge Updating

Often, new information updates our knowledge on the damage caused by environmental degradation, leading to environmental regulation being adjusted. One example is the 1988 report by the NASA Ozone Trend Panel. It accelerated by a year the implementation of the regulation agreed in the Montreal Protocol. In this section, we examine how to modify a bankable permit system when we take into account this type of uncertainty.

Assume that the total allowable pollution emissions are revised intermittently. The process is stochastic and given by a piecewise deterministic process. Let each of the events identify with a sequence  $\omega = (t_k, s_k)_{k=0}^{\infty} \subset [0, T) \times \mathbb{R}_{++}$ , where  $t_k$  is the time when a "mode" changes with  $t_0 = 0$ and  $s_k$  is the pollution target that is effective over the time interval  $[t_k, t_{k+1})$ . For expositional simplicity, we assume an infinite planning horizon  $(T = \infty)$  and the following stationarity:

Assumption 3: (a) The interest rates are constant over time, i.e., r(t) = r > 0 for all  $t \ge 0$ . (b) The goal of pollution emission controls is constant in the same mode, i.e.,  $X^*(t) = s_k$  on  $t \in [t_k, t_{k+1})$ . (c) The abatement cost function of firm i and the associated social cost function are time independent and denoted by  $C^i(x)$ , i = 1, 2, ..., n and C(X), respectively.

In addition, we assume that the knowledge converges:

**Assumption 4**:  $\lim_{k\to\infty} s_k > 0$  holds for each sample path.

Let  $E_t$  be the conditional expectation operator at time t derived from the underlying probability space of the stochastic process under consideration. The shadow price of emission control in (20) is rewritten as:

$$p^{*}(t) := -C_X(X^{*}(t)) = -C_x^{i}[x_i^{*}(X^{*}(t))].$$
(32)

Note that  $p^*(t)$  is now a random variable and is right continuous on each sample path.

First, we consider an individual firm's cost minimization problem under the implementation of a bankable permit system with a permit bank  $(\infty, X^*(t), (B_i)_{i=1}^n)$ . The problem is:

$$\min_{x(\cdot)\geq 0, y(\cdot)} E_0 \left[ \liminf_{T\to\infty} \int_0^T \{ C^i[x(t)] + p(t)y((t)) \} e^{-rt} dt \right]$$
subject to  $\dot{Q}(t) = rQ(t) + p(t) \left[ y(t) - x(t) \right],$ 

$$Q(0) = p^*(0)B_i, E_0[\liminf_{T\to\infty} Q(T)e^{-rT}] \geq 0,$$
(33)

where  $x(\cdot)$  and  $y(\cdot)$  are in the class of closed-loop controls. When we fix a sample path, the balance equation in the constraint is solved as:

$$Q(T)e^{-rT} - Q(0) = \int_0^T p(t) \left[ y(t) - x(t) \right] e^{-rt} dt$$

Using this equation to cancel out y(t) in the objective functional in (33), we have:

$$\min_{x(\cdot) \ge 0, y(\cdot)} E_0 \left[ \liminf_{T \to \infty} \left\{ \int_0^T \{ C^i[x(t)] + p(t)x((t)) \} e^{-rt} dt + \left( Q(T)e^{-rT} - Q(0) \right) \right\} \right].$$

By choosing the optimal path of y(t) to satisfy the no-Ponzi game condition with equality, the problem is reduced to:

$$\min_{x(\cdot)\geq 0, y(\cdot)} E_0\left[\liminf_{T\to\infty} \int_0^T \{C^i[x(t)] + p(t)x(t)\}e^{-rt}dt\right].$$

As there is no state variable, the equilibrium path of pollution emissions,  $x(t) = x_i^e(t)$ , satisfies:

$$p(t) = -C_x^i \left[ x_i^e(t) \right].$$
(34)

Then, we have the following lemma:

**Lemma 2** : Assume that the targeted pollution levels obey a piecewise deterministic process and that Assumptions 1–3 hold. A bankable permit system with a permit bank  $(\infty, X^*, (B_i)_{i=1}^n)$  achieves cost-effective pollution control only if the equilibrium price of permits is given by (32).

**Proof.** The statement follows from (32), (34), and the strict convexity of  $C^{i}(x)$ .

The next step is the analysis of a bankable permit system *without* a permit bank, as in the following proposition:

**Proposition 5** : Assume that the target pollution levels obey a piecewise deterministic process and that Assumptions 1–4 hold. A bankable permit system  $(\infty, X^*, (B_i)_{i=1}^n, a)$  achieves cost-effective pollution control only if the permit interest rate is set equal to the interest rate (a(t) = r) and the permits are revalued when the pollution target is revised with the rate:

$$\frac{\lim_{s \nearrow t} p^*(s)}{p^*(t)}$$

where  $p^*(t)$  is the shadow price of permits defined by (32).

**Proof.** We rewrite the balance equation in the constraint in (33) in terms of permits. We define that  $\mathcal{B}(t) = Q(t)/p^*(t)$ . Then,  $\mathcal{B}(t)$  is right continuous and jumps from  $Q(t)/\lim_{s \nearrow t} p^*(s)$  to  $Q(t)/p^*(t)$  when the mode changes. For each sample path, for  $t \in (t_k, t_{k+1}]$ , we have:

$$\mathcal{B}(t) = \frac{\lim_{s \nearrow t} p^*(s)}{p^*(t)} \left[ \mathcal{B}(t_k) + \int_{t_k}^t r \mathcal{B}(s) + \left[ y(t) - x(t) \right] dt \right],$$

with the initial value  $\mathcal{B}(0) = B_i$ . The no-Ponzi game condition is given by  $E_0[\liminf_{T\to\infty} \mathcal{B}(T)e^{-rT}] \ge 0$ , which is equivalent to the one in (33) because  $\lim_{t\to\infty} p^*(t)$  exists by Assumption 5. Therefore, under implementation of the bankable permit system, the cost minimization problem for the individual firm *i* is equivalent to (33). Then, we can apply Lemma 2.

The proposition shows that further operation is required for a bankable permit system when the pollution control goal is modified. As environmental regulation tends to strengthen, the revision indicates the devaluation of permits. In practice, many tradable permit systems, such as the US Acid Rain Program, peg the permit interest rates to zero and do not have a devaluation schedule in each phase. Compared with a cost-effective allocation of pollution emissions, the zero interest rule leads permit holders to use permits earlier, whereas the lack of a devaluation schedule leads them to save more permits. Therefore, the total effect is ambiguous in theory.

## 5 Concluding Remarks

Intertemporal flexibility of the emissions trading markets allows firms to minimize their abatement costs over time. However, the cost minimization problem implies that emissions are initially higher and then lower when there is no return for holding permits, as in the existing programs. Therefore, banking and borrowing is not necessarily socially optimal. Instead, it may result in higher total social damages. We uncover the source of this market failure in the intertemporal trading of permits and find that a tradable and bankable permit system lacks the intertemporal arbitrage condition. Thus, the regulatory authority needs to exogenously set the appropriate permit interest rates. However, this raises the problem of information collection.

This study analyzes the intertemporal allocation of tradable permits including the founding of a bank of permits. In the proposed permit bank system, the holder of permits can deposit their permits and earn the interest evaluated in a market. This type of tradable permit system can achieve a cost-effective outcome for pollution control. The permit bank moves the determination of the permit interest rate from the regulatory authority to a market. However, the equilibrium path is indeterminate. Thus, it is ambiguous whether a permit bank system achieves the target pollution level. To ensure the target level is achieved, the regulatory authority needs to set the appropriate permit interest rates.

Overall, none of the mechanisms can implement an optimal outcome without additional instruments of permit interest rates. One key implication in this study might be that Tinbergen's policy principle should be applied to an intertemporal economy: at least as many policy instruments are required as policy objectives ([19]). Here, the policy objectives are to control the abatement level of pollution at each point in time. Therefore, the policy instruments should have the same Lebesgue measure as the objectives in a continuous time model.

Finally, the results in this paper provide two key implications for the actual design of a tradable permit system. The first implication is related to the degree of the environment problems in the temporal dimension. The critical environmental problems are related to the concepts of thresholds and regime shifts of composite systems with extended consequences. If the state is near a threshold and on the verge of a regime shift, it would be safer to apply a tradable permit system with shortlife permits.<sup>7</sup> One example is the control of acidic compounds such as sulfur dioxide, the annual

<sup>&</sup>lt;sup>7</sup>Early warning of regime shifts was studied by Brock and Carpenter [3] and Brock et al. [4].

acid depositions of which have been exceeding critical loads for long periods ([5]). Therefore, the intertemporal term needs to be short, unlike the case in the US sulfur dioxide allowance program. Meanwhile, we may be able to use a long-life permit, avoiding the burden of information collection, when greenhouse gas emissions are the policy target. This is because greenhouse gases are assumed to affect society very slowly and with a huge time lag compared with the practical planning horizon of the policy scheme. Thus, the socially optimal marginal damages are almost constant in discounted terms and, therefore, no interest should be yielded on bank accounts.

The other implication of our paper is related to transaction costs. The *expost* studies show that higher administrative, implementation, and transaction costs might cause small numbers of transactions (Organization for Economic Co-operation and Development, [16]). Therefore, it is important to implement a simple mechanism in practice. We could argue that it is important to devise the intertemporal term of permits to be as short as possible, considering the difficulty of setting the appropriate permit interest rates over long terms. However, there is caution from the notable experience of the Wisconsin Fox River Water Permits (1981–1986). In six years, there has been only one trade. Several restrictions made the transfer of pollution rights a lengthy process. While the life of the permit was five years, it seems to have been too short to activate the permit market. (See Hahn [8].) This extreme case indicates that while a shorter expiration time is recommended in theory, there may be a tradeoff between the amount of transactions and the choice of the valid term.

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